

Macroscopic parity violating effects and $^3\text{He-A}$

G. E. Volovik

*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, FIN-02015 HUT, Finland
and L.D. Landau Institute for Theoretical Physics, Kosygin Str. 2, 117940 Moscow, Russia*

A. Vilenkin

Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155

(Received 25 May 1999; published 23 June 2000)

We discuss parity violating effects in relativistic quantum theory and their analogues in the effective field theory of superfluid $^3\text{He-A}$. A mixed axial-gravitational Chern-Simons term in the relativistic effective action and its condensed matter analogue are responsible for the chiral fermion flux along the rotation axis of the heat bath in a relativistic system and for the unusual Ω -odd dependence of the zero-temperature density of the normal component on the rotation velocity in $^3\text{He-A}$.

PACS number(s): 11.30.Er, 11.15.-q, 67.57.-z, 98.80.Cq

I. INTRODUCTION

In the modern view, relativistic quantum field theory and general relativity may be emergent phenomena arising in the low-energy corner of the quantum vacuum as effective theories of vacuum fluctuations [1,2]. Such effective theories of the collective degrees of freedom are typical for condensed matter systems. In a particular universality class of systems, the low-energy properties are very similar to those of the relativistic quantum vacuum. Superfluid $^3\text{He-A}$ is a representative of this class [3]. Lacking practically any symmetries above the superfluid transition, $^3\text{He-A}$ in the extreme limit of low energy acquires most of the symmetries known today in particle physics: (analogues of) Lorentz invariance, gauge invariance, general covariance, etc. The analogues of chiral Weyl fermions, as well as of gauge bosons and gravity field appear as fermionic and bosonic collective modes of the $^3\text{He-A}$ ground state. Such conceptual similarity between the condensed matter of this class and the quantum vacuum makes $^3\text{He-A}$ an ideal laboratory for simulating relativistic field theory effects in high-energy physics and cosmology.

Parity violation is one of the fundamental properties of the quantum vacuum. This effect is strong at high energy of the order of electroweak scale, but is almost imperceptible in low-energy physics. For example, Leggett's suggestion to observe the macroscopic effect of parity violation using such a macroscopically coherent atomic system as superfluid $^3\text{He-B}$ is very far from realization [4,5]. On the other hand, an analogue of parity violation exists in superfluid $^3\text{He-A}$ alongside with the related phenomena, such as chiral anomaly and macroscopic chiral currents (for a review see Refs. [5,6]). So, if we cannot investigate the macroscopic parity violating effects directly we can simulate analogous physics in $^3\text{He-A}$.

The reason why all the attributes of relativistic quantum field theory arise in $^3\text{He-A}$ can be traced to the existence of gap nodes, stable point zeros in the fermion energy spectrum. Close to each gap node the fermions necessarily become Weyl fermions and can be described by a general 2×2 matrix Hamiltonian

$$\mathcal{H} = e_a^i (p_i - p_{0i}) \tau^a. \quad (1)$$

Here the three vectors e_a^i play the role of a dreibein field which gives rise to the effective gravity field, while the position of the node in the momentum space p_{0i} plays the role of the effective electromagnetic field. These fields are dynamical and the low-energy dynamics of these collective modes is governed by the effective action, obtained by integration over the fermionic degrees of freedom, which is in complete analogy with the effective gravitational and electromagnetic actions introduced by Sakharov [2] and Zeldovich [7], respectively. If the main contribution to the integral comes from the low-energy "relativistic" fermions, the effective action automatically adopts the gauge invariance and general covariance of the low-energy fermionic Lagrangian.

The energy spectrum of fermions in $^3\text{He-A}$ is

$$\hat{H}_A = \tau^3 \frac{p^2 - p_F^2}{2m^*} + \frac{\Delta_0}{p_F} [\tau^1 \mathbf{p} \cdot \hat{\mathbf{e}}_1 + \tau^2 \mathbf{p} \cdot \hat{\mathbf{e}}_2], \quad (2)$$

$$E^2(\mathbf{p}) = \hat{H}_A^2 = \left(\frac{p^2}{2m^*} - \frac{p_F^2}{2m^*} \right)^2 + \frac{\Delta_0^2}{p_F^2} (\mathbf{p} \times \hat{\mathbf{l}})^2. \quad (3)$$

Here Δ_0 is the amplitude of the gap; m^* is the effective mass of the fermionic quasiparticles in the normal Fermi liquid, where $\Delta_0 = 0$ and the spectrum is quadratic: $E(\mathbf{p}) = (p^2 - p_F^2)/2m^*$; p_F is the Fermi momentum of the normal Fermi liquid; $\hat{\mathbf{l}}$ is a unit vector which determines the direction of the orbital angular momentum of the Cooper pairs; $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are unit mutually orthogonal vectors with $\hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2 = \hat{\mathbf{l}}$. This spectrum contains two gap nodes, at $\mathbf{p}_0 = \pm p_F \hat{\mathbf{l}}$, and close to each of these nodes the "relativistic" equation (1) is approached.

The transverse modes related with the dynamics of the $\hat{\mathbf{l}}$ vectors are Goldstone bosons known as orbital waves. Since in the relativistic domain the momentum shift $\mathbf{A}^{\text{eff}} = p_F \hat{\mathbf{l}}$ plays the role of the effective electromagnetic field acting on quasiparticles, the orbital waves represent photons and Eq. (1) can be rewritten as

$$\mathcal{H} = e_a^i (p_i - e A_i^{\text{eff}}) \tau^a \quad (4)$$

with $e = \pm 1$.

The fermionic quasiparticles living in the vicinity of opposite nodes have opposite chirality: The left-handed particles have positive “charge,” $e_L = +1$, with respect to the effective field $\mathbf{A}^{\text{eff}}$, i.e., they are concentrated near $\mathbf{p}_0 = +p_F \hat{\mathbf{l}}$. The negatively charged quasiparticles, i.e., those near $\mathbf{p}_0 = -p_F \hat{\mathbf{l}}$, are right-handed fermions, $e_R = -1$ [6]. The Pauli matrices in ${}^3\text{He-A}$, τ^a , which play the role of spin of the fermion and thus determine its chirality, are actually defined in the Bogoliubov-Nambu particle-hole space and thus describe the Bogoliubov spin. On the contrary, the ordinary spin of the ${}^3\text{He}$ atoms, which is not introduced in Eq. (2) being irrelevant for our consideration, corresponds to the weak isospin and gives rise to the effective SU(2) gauge field.

II. AXIAL ANOMALY IN ${}^3\text{He-A}$

Massless chiral fermions give rise to a number of anomalies in the effective action. The advantage of ${}^3\text{He-A}$ is that this system is complete: not only the “relativistic” infrared regime is known, but also the behavior in the ultraviolet “nonrelativistic” (or “trans-Planckian”) range is calculable, at least in principle. Since there is no need for a cutoff, all subtle issues of the anomaly can be resolved on physical grounds. The measured quantities related to the anomalies depend on the correct order of imposing limits, i.e., on what parameters of the system tend to zero faster: temperature T , external frequency ω , inverse quasiparticle lifetime due to collisions with thermal fermions $1/\tau$, inverse volume, the distance ω_0 between the energy levels of fermions, etc. All this is very important for the $T \rightarrow 0$ limit, where τ is formally infinite. An example of the crucial difference between the results obtained using different limiting procedures is the so called “angular momentum paradox” in ${}^3\text{He-A}$, which is also related to the anomaly: The orbital momentum of the fluid at $T = 0$ differs by several orders of magnitude, depending on whether the limit is taken while keeping $\omega\tau \rightarrow 0$ or $\omega\tau \rightarrow \infty$. The “angular momentum paradox” in ${}^3\text{He-A}$ has possibly a common origin with the anomaly in the spin structure of hadrons [8].

In the spatially inhomogeneous case there is another important parameter $\omega_0\tau$, where ω_0 is the distance between the localized energy levels. The gapless fermions in ${}^3\text{He-A}$ lead to the momentum exchange between the superfluid vacuum and the normal component of the liquid (the gas of fermionic quasiparticles). This exchange is mediated by the texture of the $\hat{\mathbf{l}}$ field: $\dot{\mathbf{p}} = (p_F^3/2\pi^2)\hat{\mathbf{l}}(\partial_t \hat{\mathbf{l}} \cdot (\nabla \times \hat{\mathbf{l}}))$. Since the dictionary for translation to the language of relativistic theory reads $\mathbf{A}^{\text{eff}} \equiv p_F \hat{\mathbf{l}}$, one obtains

$$\dot{\mathbf{p}} = p_F \hat{\mathbf{l}} \dot{n}, \quad (5)$$

$$\dot{n} = \frac{1}{2\pi^2} \partial_t \mathbf{A}^{\text{eff}} \cdot (\nabla \times \mathbf{A}^{\text{eff}}). \quad (6)$$

Equation (6) is nothing but the Adler-Bell-Jackiw axial anomaly equation [9], describing the production of chiral fermions from the quantum vacuum due to the spectral flow through the gap nodes. The relevant fermionic charge, which is produced due to the chiral anomaly in ${}^3\text{He-A}$, is the linear momentum $p_F \hat{\mathbf{l}}$ of fermionic quasiparticle: It is the rate of the momentum production, which is measured in experiments on the dynamics of the vortex textures as an extra force acting on a moving texture [10].

It appears, however, that Eqs. (5),(6) are valid only in the limit of continuous spectrum, i.e., when the distance ω_0 between the energy levels of fermions in the texture is much smaller than the inverse quasiparticle lifetime: $\omega_0\tau \ll 1$. The spectral flow completely disappears in the opposite case $\omega_0\tau \gg 1$, because the spectrum becomes effectively discrete. As a result, the force acting on a vortex texture differs by several orders of magnitude for the cases $\omega_0\tau \ll 1$ and $\omega_0\tau \gg 1$. The parameter $\omega_0\tau$ is regulated by temperature. The Adler-Bell-Jackiw equation was experimentally confirmed in experiments with rotating ${}^3\text{He-A}$ performed in the limit $\omega_0\tau \ll 1$ [10,11]. The transfer from the axial anomaly regime $\omega_0\tau \ll 1$ to the regime of the suppressed spectral flow $\omega_0\tau \gg 1$ has been observed for ${}^3\text{He-B}$ vortices, whose dynamics is governed by the similar spectral flow in the vortex core [10,11].

The chiral anomaly also leads to the chiral current, which is proportional to $\mathbf{A}^{\text{eff}} \cdot (\nabla \times \mathbf{A}^{\text{eff}})$. This current has been proved to exist: it leads to an observed instability of the superflow [12,11]. The same instability of the system of right-handed electrons towards production of a hypermagnetic field was discussed by Joyce and Shaposhnikov [13] in relation to the generation of a primordial magnetic field.

III. MIXED AXIAL-GRAVITATIONAL CHERN-SIMONS TERM IN THE EFFECTIVE ACTION

A. Parity violating current

Here we discuss another particular case of correspondence between relativistic field theory, in which the chiral anomaly problem can be mapped to the angular momentum paradox in ${}^3\text{He-A}$. It involves macroscopic parity violating effects in a rotating system with chiral fermions, discussed in Ref. [14]. The angular velocity of rotation $\boldsymbol{\Omega}$ defines the preferred direction of polarization, and right-handed fermions move in the direction of their spin. As a result, such fermions develop a current parallel to $\boldsymbol{\Omega}$. Similarly, left-handed fermions develop a current antiparallel to $\boldsymbol{\Omega}$. The corresponding current density was calculated in Ref. [14], assuming thermal equilibrium at temperature T and chemical potential of the fermions μ . For right-handed fermions, it is given by

$$\mathbf{j} = \left(\frac{T^2}{12} + \frac{\mu^2}{4\pi^2} \right) \boldsymbol{\Omega}. \quad (7)$$

The current $\mathbf{j}$ is a polar vector, while the angular velocity $\boldsymbol{\Omega}$ is an axial vector, and thus Eq. (7) violates the reflectional symmetry. We are going to show that this current gives rise to what can be called mixed axial-gravitational Chern-

Simons terms in the effective action and that equivalent terms do exist in the thermodynamic potential of $^3\text{He-A}$.

If the current (7) is coupled to a gauge field A^ν , the appropriate term in the Lagrangian density is

$$L = e\mathbf{A} \cdot \mathbf{j}/c^2, \quad (8)$$

where e is the gauge coupling. The correspondence between field theory and $^3\text{He-A}$ is achieved by replacing the gauge field and the metric by appropriate $^3\text{He-A}$ observables [6]. Here we shall start from the opposite end and derive the $\mathbf{\Omega}$ -dependent contribution to the free energy. We shall then show that it is equivalent to Eq. (8). We note that to establish the correspondence, the free energy should be expressed in a covariant and gauge invariant form and should not contain any material parameters, such as “speed of light.” Then it can be equally applied to both systems, standard model and $^3\text{He-A}$.

B. Orbital angular momentum and free energy

Let us consider a stationary liquid $^3\text{He-A}$ in a vessel rotating with angular velocity $\mathbf{\Omega}$ at a nonzero temperature. We assume a spatially homogeneous vector $\hat{\mathbf{I}}$ oriented along the rotation axis. In $^3\text{He-A}$ this can be achieved in the parallel-plane geometry, while in the layered oxide superconductor Sr_2RuO_4 , which is believed to be a triplet superconductor with a $^3\text{He-A}$ -like order parameter, the $\hat{\mathbf{I}}$ -vector is always fixed along the normal to the layers [15]. The superfluid component (vacuum) is assumed to be at rest, while the normal component—the heat bath of thermal fermions—circulates in the plane perpendicular to $\hat{\mathbf{I}}$ with the velocity $\mathbf{v}_n = \mathbf{\Omega} \times \mathbf{r}$.

The value of the angular momentum of a rotating $^3\text{He-A}$ has been a subject of long-standing controversy (for a review see Refs. [5,6]). Different methods for calculating the angular momentum give results that differ by many orders of magnitude. The result is also sensitive to the boundary conditions, since the angular momentum in the liquid is not necessarily the local quantity, and to whether the state is strictly stationary or has a small but finite frequency. This is often referred to as the angular momentum paradox. The paradox is related to the axial anomaly induced by chiral quasiparticles and is now reasonably well understood.

According to Kita conjecture [16], which was supported by his numerical calculations, the total angular momentum of the liquid with $\hat{\mathbf{I}} = \text{const}$ corresponds to the following angular momentum density:

$$\mathbf{L}(T) = \frac{\hbar}{2} \hat{\mathbf{I}} n_{\parallel}(T), \quad (9)$$

where $n_{\parallel}(T)$ is the temperature dependent density of the superfluid component when it flows along $\hat{\mathbf{I}}$. We recall that the current of the ^3He atoms has two contributions in the superfluid state:

$$\mathbf{J} = n\mathbf{v}_s + \mathbf{J}_q, \quad \mathbf{J}_q = \sum_{\mathbf{p}} \frac{\mathbf{p}}{m_3} f(\mathbf{p}). \quad (10)$$

The first term is the current transferred by the superfluid vacuum moving with velocity $\mathbf{v}_s$; n is the particle density of ^3He liquid. The second term is the contribution of quasiparticles, where $f(\mathbf{p})$ is the quasiparticle distribution function and m_3 is the bare mass of ^3He atom. In equilibrium one has

$$f(\mathbf{p}) = \left(\exp \frac{\tilde{E}(\mathbf{p}) - \mathbf{p} \cdot \mathbf{v}_n}{T} + 1 \right)^{-1}, \quad \tilde{E} = E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}_s, \quad (11)$$

where $\tilde{E}$ is the Doppler shifted energy of quasiparticle, when the superfluid vacuum is moving. In the linear in velocity regime one obtains $J_{qi} = n_{nik}(v_{ni} - v_{sk})$, where n_{nik} is the so called density of the normal component. In $^3\text{He-A}$ the normal component density is a uniaxial tensor with the anisotropy axis along $\hat{\mathbf{I}}$: the density involved in the normal motion depends on the orientation of $\mathbf{v}_n - \mathbf{v}_s$ with respect to $\hat{\mathbf{I}}$ [5]. The tensor of the superfluid density is $n_{sik} = n\delta_{ik} - n_{nik}$. At $T=0$ one has $n_{\parallel}(0) = n$.

The contribution of Eq. (9) to the free energy density is

$$F = -\mathbf{\Omega} \cdot \mathbf{L}(T) = -\mathbf{\Omega} \cdot \mathbf{L}(0) + \frac{\hbar}{2} (\mathbf{\Omega} \cdot \hat{\mathbf{I}}) n_{\parallel}(T). \quad (12)$$

The first (zero-temperature) term on the right-hand side of Eq. (12) has no analogue in field theory, and we disregard it in what follows. The second term comes from chiral quasiparticles, which comprise the normal component. Its longitudinal density $n_{\parallel} = n - n_{\perp}$ at $T \ll T_c$ is obtained from Eqs. (10),(11) [5]:

$$n_{\parallel} \approx \frac{m^*}{3m_3} p_F^3 \frac{T^2}{\Delta_0^2}. \quad (13)$$

Kita [16] obtained his result, Eq. (9), for the simplest case when $m^* = m_3$, i.e., the interactions which renormalize the quasiparticle mass in normal Fermi liquid were neglected. We expect, however, that Eq. (9) is insensitive to the details of interaction (see below) and shall concentrate on this case.

C. Effective Chern-Simons action at $T \neq 0$, $\mu = 0$

Let us now use the dictionary for translating Eq. (12) to the language of relativistic theories. If one chooses the reference frame rotating with the normal component, then the effective gauge field and the effective metric “seen” by the fermionic quasiparticles are

$$\mathbf{A} \equiv \mathbf{A}^{\text{eff}} = p_F \hat{\mathbf{I}}, \quad (14)$$

$$g^{ik} = c_{\perp}^2 (\hat{e}_1^i \hat{e}_1^k + \hat{e}_2^i \hat{e}_2^k) + c_{\parallel}^2 \hat{I}^i \hat{I}^k - v_n^i v_n^k, \quad g^{0i} = v_n^i, \quad (15)$$

$$c_{\perp} = \frac{\Delta_0}{p_F}, \quad c_{\parallel} = \frac{p_F}{m_3}. \quad (16)$$

They are obtained by linearizing Eq. (2) in the vicinity of the nodes. The mixed components of the metric tensor $g^{0i} = v_n^i$ come from the Doppler shift in the comoving frame. The

angular velocity is expressed through $\mathbf{v}_n$ as $\mathbf{\Omega} = (1/2)\nabla \times \mathbf{v}_n$ and thus is proportional to the effective gravimagnetic field [17]

$$\mathbf{B}_g = \nabla \times \mathbf{g} = 2 \frac{\mathbf{\Omega}}{c_\perp^2}, \quad \mathbf{g} \equiv g_{0i} = \frac{v_{ni}}{c_\perp^2}. \quad (17)$$

Here, we have made the following assumptions: (i) $v_n \ll c_\perp$ everywhere in the vessel, i.e., the counterflow velocity $\mathbf{v}_n - \mathbf{v}_s$ is smaller than the pair-breaking critical velocity $c_\perp = \Delta_0/p_F$ (the transverse “speed of light”). This means that there is no region in the vessel where particles can have negative energy (ergoregion). Effects caused by the ergoregion in superfluids are discussed in Ref. [18]. (ii) Rotation velocity is so small that there are no vortices in the container. This is typical for superfluid ^3He , where the critical velocity for nucleation of vortices is comparable to the pair-breaking velocity. Even in the geometry when the $\hat{\mathbf{l}}$ vector is not fixed, the observed critical velocity in $^3\text{He-A}$ was found to reach 0.5 rad/sec. For the geometry with fixed $\hat{\mathbf{l}}$, it should be comparable with the critical velocity in $^3\text{He-B}$. (iii) We approach the $T \rightarrow 0$ limit in such a way that there is still a reference frame specified by the thermal bath of fermionic excitations, which rotates together with the container in equilibrium. This corresponds to the case when the condition $\omega\tau \ll 1$ remains valid, despite the divergence of τ .

The translation of Eqs. (12),(13) to the relativistic language can be presented as the following term in the Lagrangian:

$$L_{\text{CS}} = \frac{e_R - e_L}{24} T^2 \mathbf{A} \cdot \mathbf{B}_g = \frac{e_R - e_L}{24} T^2 e^{ijk} A_i \nabla_j g_{k0}. \quad (18)$$

Since the parity of $^3\text{He-A}$ fermions is opposite to their charge, the contributions of the two species in $^3\text{He-A}$ are not cancelled but are added together.

Equation (18) does not contain explicitly any material parameters, such as the “speeds of light” $c_\parallel$ and $c_\perp$. Moreover, it is gauge invariant, provided that the system is in thermal equilibrium, i.e., $T = \text{const}$. Thus it can be immediately applied to the relativistic theory in a rotating frame in Minkowski space, where $e^{ijk} \nabla_j g_{k0} = 2\mathbf{\Omega}^i/c^2$, with c being the speed of light. Note that the angular velocity $\mathbf{\Omega}$ retains the same meaning in relativistic theory. It is the angular velocity which appears in the distribution function of thermal fermions. With the aid of Eq. (7) it is easily verified that Eq. (18) is equivalent to the Lagrangian density (8), if we set $\mu = 0$, $e_R = e$ and $e_L = 0$.

Equation (18) is not Lorentz invariant, but this is not important here because the existence of a heat bath does violate the Lorentz invariance, since it provides a distinguished reference frame. To restore the Lorentz invariance and also the general covariance one must introduce the four-velocity (u^μ) and/or four-temperature (β^μ) of the heat bath fermions. But this is not necessary since the unification of the chiral effects in the two systems has been achieved already at this level.

We next consider the effect of a finite chemical potential, which in our case corresponds to the superfluid-normal coun-

terflow. We shall assume a superflow along the axis of the cylinder and consider its effect in the presence of rotation.

D. Nonzero μ vs nonzero axial counterflow

A superfluid-normal counterflow velocity along $\hat{\mathbf{l}}$ produces a Doppler shift, $\tilde{E} = E + \mathbf{p} \cdot (\mathbf{v}_s - \mathbf{v}_n)$. In the vicinity of the two nodes one has $\tilde{E} \approx E \pm p_F \hat{\mathbf{l}} \cdot (\mathbf{v}_s - \mathbf{v}_n)$, which means that the counterflow enters the Lagrangian for fermionic quasiparticles in $^3\text{He-A}$ in the same way as the chemical potentials for relativistic chiral fermions [11]

$$\mu_R = -\mu_L \equiv -p_F \hat{\mathbf{l}} \cdot (\mathbf{v}_s - \mathbf{v}_n). \quad (19)$$

That is why the energy stored in the system of chiral fermions $A = F - \mu_R N_R - \mu_L N_L$, and the energy of the counterflow along the $\hat{\mathbf{l}}$ vector $A = F - m_3 (\mathbf{J}_q \cdot \hat{\mathbf{l}}) (\hat{\mathbf{l}} \cdot (\mathbf{v}_n - \mathbf{v}_s))$, are described by the same thermodynamic potential. At $T = 0$ it is

$$A = -\frac{\sqrt{-g}}{12\pi^2} (\mu_R^4 + \mu_L^4) \equiv -\frac{m_3 p_F^3}{12\pi^2 c_\perp^2} ((\hat{\mathbf{l}} \cdot (\mathbf{v}_s - \mathbf{v}_n))^4). \quad (20)$$

Variation of Eq. (20) with respect to $\mathbf{v}_n$ gives the mass current along the $\hat{\mathbf{l}}$ vector produced by the fermionic quasiparticles, which at $T = 0$ occupy the energy levels with $\tilde{E} < 0$:

$$m_3 J_{q\parallel} = -\frac{dA}{dv_{n\parallel}}. \quad (21)$$

This shows that in the presence of a superflow with respect to the heat bath the normal component density is nonzero even in the limit $T \rightarrow 0$ [19]:

$$n_{n\parallel}(T \rightarrow 0) = \frac{dJ_{q\parallel}}{dv_{n\parallel}} = \frac{p_F^3}{3\pi^2 c_\perp^2} (\hat{\mathbf{l}} \cdot (\mathbf{v}_s - \mathbf{v}_n))^2. \quad (22)$$

Now we can check how general is the Kita conjecture, Eq. (9). Let us assume that Eq. (12) remains valid even if the normal component at $T = 0$ is added. Then from Eq. (22) it follows that the mixed Chern-Simons term becomes

$$L_{\text{CS}} = \frac{e_R \mu_R^2 - e_L \mu_L^2}{8\pi^2} \mathbf{A} \cdot \mathbf{B}_g. \quad (23)$$

This term is equivalent to the Lagrangian density (7),(8) in the appropriate limit ($T = 0$, $e_R = e$, $e_L = 0$, $\mu_R = \mu$, $\mu_L = 0$), which thus confirms the above assumption.

IV. DISCUSSION

The form of Eq. (23) is similar to that of the induced Chern-Simons term

$$\tilde{L}_{\text{CS}} = \frac{\mu}{2\pi^2} \mathbf{A} \cdot \mathbf{B} \quad (24)$$

with $\mathbf{B} = \nabla \times \mathbf{A}$, which has been extensively discussed both in the context of chiral fermions in relativistic theory [20–23]

and in $^3\text{He-A}$ [11]. The main difference between Eqs. (24) and (23) is that $\mathbf{B}_g = \nabla \times \mathbf{g}$ is the gravimagnetic field, rather than the magnetic field $\mathbf{B}$ associated with the potential $\mathbf{A}$. Hence the name “mixed Chern-Simons term.”

The parity-violating currents (7) could be induced in turbulent cosmic plasmas and could play a role in the origin of cosmic magnetic fields [24]. The corresponding liquid Helium effects are less dramatic but may in principle be observable.

Although the mixed Chern-Simons terms have the same form in relativistic theories and in $^3\text{He-A}$, their physical manifestations are not identical. In the relativistic case, the electric current of chiral fermions is obtained by variation with respect to $\mathbf{A}$, while in $^3\text{He-A}$ case the observable effects are obtained by variation of the same term but with respect to $^3\text{He-A}$ observables. For example, the expression for the current of ^3He atoms is obtained by variation of Eq. (23) over $\mathbf{v}_n$. This leads to an extra particle current along the rotation axis, which is odd in Ω :

$$\Delta \mathbf{J}_q(\Omega) = \frac{p_F^3}{\pi^2} \hat{\mathbf{l}} (\hat{\mathbf{l}} \cdot (\mathbf{v}_s - \mathbf{v}_n)) \frac{\hat{\mathbf{l}} \cdot \Omega}{m_3 c_\perp^2}. \quad (25)$$

Equation (25) shows that there is an Ω odd contribution to the normal component density at $T \rightarrow 0$ in $^3\text{He-A}$:

$$\Delta n_{\parallel}(\Omega) = \frac{\Delta J_q(\Omega)}{v_{\parallel} - v_{\parallel}} = \frac{p_F^3}{\pi^2} \frac{\hat{\mathbf{l}} \cdot \Omega}{m_3 c_\perp^2}. \quad (26)$$

The sensitivity of the normal component density to the direction of rotation is the counterpart of the parity violation effects in relativistic theories with chiral fermions. It should be noted though that, since $\hat{\mathbf{l}}$ is an axial vector, the right-hand sides of Eqs. (25) and (26) transform, respectively, as a polar vector and a scalar, and thus (of course) there is no real parity violation in $^3\text{He-A}$. However, a nonzero expectation value of the axial vector of the orbital angular momentum

$\mathbf{L} = (\hbar/2)n_{\parallel}(T)\hat{\mathbf{l}}$ does indicate a *spontaneously* broken reflectional symmetry, and an internal observer “living” in a $^3\text{He-A}$ background with a fixed $\hat{\mathbf{l}}$ would observe parity-violating effects.

The contribution (26) to the normal component density can have arbitrary sign depending on the sense of rotation with respect to $\hat{\mathbf{l}}$. This, however, does not violate the general rule that the overall normal component density must be positive: The rotation dependent current $\Delta \mathbf{J}_q(\vec{\Omega})$ was calculated as a correction to the rotation independent current in Eq. (21). This means that we used the condition $\hbar\Omega \ll m_3(v_{\parallel} - v_{\parallel})^2 \ll m_3 c_\perp^2$. Under this condition the overall normal density, given by the sum of Eqs. (26) and (22), remains positive.

The “parity” effect in Eq. (26) is not very small. The rotational contribution to the normal component density normalized to the density of the ^3He atoms is $\Delta n_{\parallel}/n = 3\Omega/m_3 c_\perp^2$ which is $\sim 10^{-4}$ for $\Omega \sim 3$ rad/s. This is within the resolution of the vibrating wire detectors.

We finally mention a possible application of our results to the superconducting Sr_2RuO_4 [15]. An advantage of using superconductors is that the particle current $\Delta \mathbf{J}_q$ in Eq. (25) is accompanied by the electric current $e\Delta \mathbf{J}_q$, and can be measured directly. An observation in Sr_2RuO_4 of the analogue of the parity violating effect that we discussed here (or of the other effects coming from the induced Chern-Simons terms [25]), would be an unquestionable evidence of the chirality of this superconductor.

ACKNOWLEDGMENTS

We thank Matti Krusius who suggested to apply our results to chiral superconductors. The work of G.V. was supported in part by the Russian Foundation for Fundamental Research and by European Science Foundation. The work of A.V. was supported in part by the National Science Foundation.

-
- [1] S. Weinberg “What is Quantum Field Theory, and What Did We Think It Is?” hep-th/9702027.
 - [2] A.D. Sakharov, Sov. Phys. Dokl. **12**, 1040 (1968).
 - [3] G.E. Volovik, Proc. Natl. Acad. Sci. USA **96**, 6042 (1999).
 - [4] A.J. Leggett, Phys. Rev. Lett. **39**, 587 (1977).
 - [5] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor and Francis, London, 1990).
 - [6] G. E. Volovik, *Exotic Properties of Superfluid ^3He* (World Scientific, Singapore, 1992); G.E. Volovik and T. Vachaspati, Int. J. Mod. Phys. B **10**, 471 (1996).
 - [7] Ya.B. Zeldovich, JETP Lett. **6**, 345 (1967).
 - [8] S. M. Troshin and N. E. Tyurin, Phys. Rev. D **55**, 1265 (1997).
 - [9] S. Adler, Phys. Rev. **177**, 2426 (1969); J.S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969).
 - [10] T.D.C. Bevan, A.J. Manninen, J.B. Cook, J.R. Hook, H.E. Hall, T. Vachaspati, and G.E. Volovik, Nature (London) **386**, 689 (1997).
 - [11] G.E. Volovik, Physica B **255**, 86 (1998).
 - [12] V.M.H. Ruutu, J. Kopu, M. Krusius, U. Parts, B. Plaçais, E.V. Thuneberg, and W. Xu, Phys. Rev. Lett. **79**, 5058 (1997).
 - [13] M. Joyce and M. Shaposhnikov, Phys. Rev. Lett. **79**, 1193 (1997).
 - [14] A. Vilenkin, Phys. Rev. D **20**, 1807 (1979); **21**, 2260 (1980).
 - [15] M. Rice, Nature (London) **396**, 627 (1998); K. Ishida *et al.*, *ibid.* **396**, 658 (1998).
 - [16] T. Kita, J. Phys. Soc. Jpn. **67**, 216 (1998).
 - [17] D. Lynden-Bell and M. Nouri-Zonoz, Rev. Mod. Phys. **70**, 427 (1998).
 - [18] A. Calogeracos and G.E. Volovik, JETP Lett. **69**, 281 (1999).
 - [19] P. Muzikar and D. Rainer, Phys. Rev. B **27**, 4243 (1983); K. Nagai, J. Low Temp. Phys. **55**, 233 (1984); G.E. Volovik, in *Helium Three*, edited by W.P. Halperin and L.P. Pitaevskii (Elsevier Science Publishers, British Vancouver, 1990), pp. 27–134.

- [20] A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).
- [21] A.N. Redlich and L.C.R. Wijewardhana, Phys. Rev. Lett. **54**, 970 (1985).
- [22] R. Jackiw and V. Alan Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999).
- [23] A.A. Andrianov, R. Soldati, and L. Sorbo, Phys. Rev. D **59**, 025002 (1999).
- [24] A. Vilenkin and D.A. Leahy, Astrophys. J. **254**, 77 (1982).
- [25] J. Goryo and K. Ishikawa, Phys. Lett. A **260**, 294 (1999); G.E. Volovik, Sov. Phys. JETP **67**, 1804 (1988).